Refresher Math Problem Set A3

(Multiplying and Dividing Fractions)

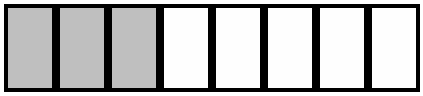
Ground Rules for Problem Set Completion

1. Present your work in a neat and organized manner. Use complete sentences whenever you are asked to make a statement.
2. SHOW YOUR WORK: Credit is awarded for all reasonable attempts, based on the work shown.
3. Make sure you answer ALL parts of problems.
4. Complete and submit ALL Problem Sets for the unit prior to taking the Unit Test.

SAMPLE PROBLEM 1 WITH SOLUTION (TO BE USED TO ANSWER PROBLEM I-A)

Question | **Solution**

1. Sketch a figure that represents the fraction $^3/_8$ by drawing a rectangle, dividing it into eight equal portions, and shading three of the eight portions.



2. Use steps similar to those in Part (1) to represent the fraction $^5/_{12}$.

3. Find the Least Common Denominator (LCD) of $^3/_8$ and $^5/_{12}$.

   a. Check to see if the largest denominator is divisible by the smaller denominator.
      - 12 is NOT divisible by 8, therefore 12 is NOT the LCD.

   b. List several multiples of each denominator. (I usually start by listing 4 or 5 multiples of the largest denominator. I can always add to the list, if needed.)

      Multiples of 12: 12, **24**, 36, 48

      Multiples of 8: 8, 16, **24**

      - 24 is the smallest number that appears in both lists, therefore 24 is the Least Common Denominator.

4. Find the sum of $^3/_8$ and $^5/_{12}$ by adding them together after using the LCD from Part (3) to find equivalent fractions.

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$$+\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}$$

$$\frac{19}{24}$$

5. Find the difference between $^3/_8$ and $^5/_{12}$ by subtracting the smaller number from the larger one after using the LCD from Part (3) to find equivalent fractions.

$$\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}$$

$$-\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$$\frac{1}{24}$$

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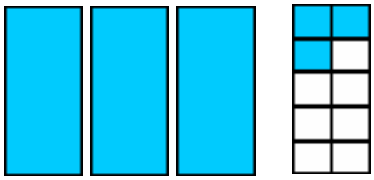
SAMPLE PROBLEM 2 WITH SOLUTION (TO BE USED TO ANSWER PROBLEM I-B)

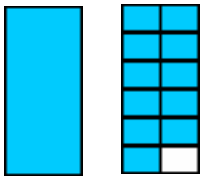
Question

1. Sketch a figure that represents the mixed number $3^{3}/_{10}$ by drawing a filled-in rectangle for each whole unit; then, drawing another rectangle, dividing it into ten equal portions, and shading three of the ten portions. [Notice that each rectangle is the same size since each represents one whole unit.]

2. Use steps similar to those in Part (1) to represent the mixed number $1^{11}/_{12}$.

Solution





3. Find the Least Common Denominator (LCD) of $^{3}/_{10}$ and $^{11}/_{12}$.

   a. Check to see if the largest denominator is divisible by the smaller denominator.
      - 12 is NOT divisible by 10, therefore 12 is NOT the LCD.

   b. List several multiples of each denominator. (I usually start by listing 4 or 5 multiples of the largest denominator. I can always add to the list, if needed.)

      Multiples of 12: 12, 24, 36, 48, **60**

      Multiples of 10: 10, 20, 30, 40, 50, **60**

      - 60 is the smallest number that appears in both lists, therefore 60 is the Least Common Denominator.

4. Find the sum of $3^{3}/_{10}$ and $1^{11}/_{12}$.

$$3\frac{3}{10} = 3\frac{3 \times 6}{10 \times 6} = 3\frac{18}{60}$$
$$+\ 1\frac{11}{12} = 1\frac{11 \times 5}{12 \times 5} = 1\frac{55}{60}$$
$$4\frac{73}{60} = 5\frac{13}{60}$$

5. Find the difference between $3^{3}/_{10}$ and $1^{11}/_{12}$.

$$3\frac{3}{10} = 3\frac{3 \times 6}{10 \times 6} = 3\frac{18}{60} = 2\frac{78}{60}$$
$$-\ 1\frac{11}{12} = 1\frac{11 \times 5}{12 \times 5} = 1\frac{55}{60} = 1\frac{55}{60}$$
$$1\frac{23}{60}$$

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I. REVIEW PROBLEMS

The problems below provide practice with skills and concepts from Problem Sets A1 and A2.

A. Sample Problem 1 on page 1 should be used as a model for answering this problem.

1. Sketch a figure that represents the fraction $^3/_{10}$ by drawing a rectangle, dividing it into ten equal portions, and shading three of the ten portions.
2. Use steps similar to those in Part (1) to represent the fraction $^1/_6$.
3. Find the Least Common Denominator (LCD) of $^3/_{10}$ and $^1/_6$.
4. Find the sum of $^3/_{10}$ and $^1/_6$, using the LCD from Part (3) to find equivalent fractions.
5. Find the difference between $^3/_{10}$ and $^1/_6$ by subtracting the smaller number from the larger one and using the LCD from Part (3) to find equivalent fractions.

B. Sample Problem 2 on page 2 should be used as a model for answering this problem.

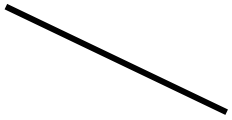
1. Sketch a figure that represents the mixed number $2^5/_8$ by drawing a filled-in rectangle for each whole unit; then, drawing another rectangle, dividing it into eight equal portions, and shading five of the eight portions.
2. Use steps similar to those in Part (1) to represent the mixed number $1^{19}/_{20}$.
3. Find the Least Common Denominator (LCD) of $^5/_8$ and $^{19}/_{20}$.
4. Find the sum of $2^5/_8$ and $1^{19}/_{20}$.
5. Find the difference between $2^5/_8$ and $1^{19}/_{20}$.

C. Add or subtract as indicated. Give answers as proper fractions or mixed numbers in lowest terms.

1. $^8/_9 + ^2/_5 + ^4/_3 =$
2. $7^6/_{11} - 3^1/_5 =$
3. $1^3/_5 + ^1/_2 + 6^9/_{20}$

4. $12\frac{1}{4} - 5\frac{9}{16}$
5. $8\frac{3}{10} + 7\frac{5}{6}$
6. $7 - 3\frac{3}{8}$

D. Measure each line to the nearest $^1/_{16}$ inch. Give your answers in lowest terms.

1. 
2.

E. Completely solve the problems below using steps (*i*), (*ii*), and (*iii*).

i. State what it is you are to find. Give your answer as a complete sentence.
ii. Solve the problem, showing your work.
iii. State the answer in a complete sentence.

1. ABC Gum stock started the day at $37^3/_8$ and dropped $2^3/_4$ points by the end of the day. What was the stock's point value at the end of the day?
2. On Monday Jim and Ilene drove $3^2/_5$ miles to the drug store, then $2^3/_4$ miles to the grocery store, and $4^7/_{10}$ miles on the trip back home. In all, how many miles did Jim and Ilene travel?
3. Larry budgets $^1/_4$ of his income for housing, $^1/_5$ for car expenses, $^1/_8$ for groceries, and $^3/_{16}$ for utilities and phone. What fraction of his income has Larry budgeted for the listed items?
4. Based on your answer to Problem 3 above, what fraction of Larry's income is left after he has budgeted for housing, car, grocery, utility, and phone expenses?

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(Multiplying and Dividing Fractions)

II. MULTIPLYING FRACTIONS & MIXED NUMBERS

As you will see, multiplying fractions is usually easier than adding and subtracting fractions since you do not need to find common denominators to multiply.

SAMPLE PROBLEM 3 DEMONSTRATES HOW TO MULTIPLY FRACTIONS.

SAMPLE PROBLEM 3 WITH SOLUTION

The Problem:

Multiply & Reduce: $^{5}/_{6} * {}^{8}/_{3} * {}^{3}/_{4}$

The Solution:

1. Multiply the numerators (top numbers) together (5 * 8 * 3 = 120).
2. Multiply the denominators (bottom numbers) together (6 * 3 * 4 = 72).
3. Write the resulting fraction: $^{120}/_{72}$.
4. Reduce to a fraction or mixed number in lowest terms: $^{120}/_{72} = 1\,^{48}/_{72} = 1\,^{2}/_{3}$

A. Multiply and reduce. Refer to Sample Problem 3 as needed. For more practice, see page 39 of Contemporary's Number Power 2 work-text.

1. $^{9}/_{10} * {}^{7}/_{13} =$
2. $^{5}/_{6} * {}^{3}/_{2} * {}^{1}/_{10} =$
3. $^{4}/_{3} * {}^{1}/_{2} * {}^{15}/_{4} * {}^{3}/_{8} =$

We can simplify the multiplication process further if we **cancel factors common to both the numerators and the denominators**. By doing so we end up with smaller numbers that are easier to work with and we eliminate the need to reduce at the end.

SAMPLE PROBLEM 4 DEMONSTRATES HOW TO CANCEL WHEN MULTIPLYING FRACTIONS.

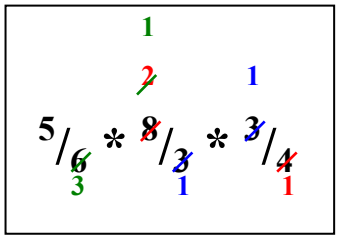
SAMPLE PROBLEM 4 WITH SOLUTION

The Problem:

Cancel, Multiply & Reduce: $^{5}/_{6} * {}^{8}/_{3} * {}^{3}/_{4}$

The Solution:

1. Cancel any factors in the numerators that also appear in the denominators. (In this case the 3 in the numerator of $^{3}/_{4}$ cancels the 3 in the denominator of $^{8}/_{3}$, the 4 in the denominator of $^{3}/_{4}$ cancels a factor of 4 from the 8 in the numerator of $^{8}/_{3}$, and the factor of 2 from the 8 in the numerator of $^{8}/_{3}$ cancels a factor of 2 from the 6 in the denominator of $^{5}/_{6}$.)



2. Multiply the resulting numerators (top numbers) together (5 * 1 * 1 = 5).
3. Multiply the resulting denominators (bottom numbers) together (3 * 1 * 1 = 3).
4. Write the resulting fraction or mixed number: $^{5}/_{3} = 1\,^{2}/_{3}$.

B. Cancel where possible, then multiply and reduce. Refer to Sample Problem 4 as needed. For more practice, see pages 40 and 41 of Contemporary's Number Power 2 work-text.

1. $^{3}/_{10} * {}^{5}/_{12} =$
2. $^{4}/_{15} * {}^{45}/_{24} =$
3. $^{5}/_{6} * {}^{12}/_{15} * {}^{30}/_{8} =$

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To this point we have consider multiplication that involves just fractions. Often, however, we need to solve problems that involve the multiplication of fractions, whole numbers, and mixed numbers**.**

This process is not all that different from fraction multiplication. In fact, there is only one additional step we must do – **first change all whole and mixed numbers to improper fractions.**

SAMPLE PROBLEM 5 DEMONSTRATES MULTIPLICATION INVOLVING FRACTIONS, WHOLE NUMBERS, AND MIXED NUMBERS.

SAMPLE PROBLEM 5 WITH SOLUTION

The Problem:

Multiply & Reduce, canceling when possible: $1\,{}^{1}/_{6} * 5 * 2\,{}^{4}/_{7} * {}^{9}/_{10}$

The Solution:

1. Convert all whole numbers and mixed numbers to improper fractions.

$$1\,{}^{1}/_{6} * 5 * 2\,{}^{4}/_{7} * {}^{9}/_{10} =$$
$${}^{7}/_{6} * {}^{5}/_{1} * {}^{18}/_{7} * {}^{9}/_{10} =$$

2. Cancel any factors in the numerators that also appear in the denominators.

$$\frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{6}}} * \frac{\overset{1}{\cancel{5}}}{1} * \frac{\overset{9}{\cancel{18}}}{\underset{1}{\cancel{7}}} * \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{10}}}$$

3. Multiply the resulting numerators together ($1 * 1 * 9 * 3 = 27$).
4. Multiply the resulting denominators together ($2 * 1 * 1 * 1 = 2$).
5. Write the resulting fraction or mixed number: ${}^{27}/_{2} = 13\,{}^{1}/_{2}$.

C. Convert whole numbers and mixed numbers to improper fractions, cancel where possible, then multiply and reduce. Refer to Sample Problem 5 as needed. For more practice, see pages 42 and 43 of Contemporary's Number Power 2 work-text.

1. $4 * {}^{5}/_{12} * 1\,{}^{3}/_{5} =$ 2. ${}^{3}/_{8} * 2\,{}^{5}/_{6} * 3 =$ 3. $8\,{}^{1}/_{2} * 1\,{}^{2}/_{5} * 3\,{}^{7}/_{8} =$

III. DIVIDING FRACTIONS & MIXED NUMBERS

Division is defined as multiplying by the reciprocal. (Two numbers are reciprocals if their product is 1. For example, ${}^{3}/_{5}$ and ${}^{5}/_{3}$ are reciprocals since ${}^{3}/_{5} * {}^{5}/_{3} = 1$.) Thus, to divide fractions we simply **invert the divisor** (the number we are dividing by) **and multiply**.

SAMPLE PROBLEM 6 (NEXT PAGE) DEMONSTRATES DIVISION OF FRACTIONS.

As with multiplication problems, the first step when simplifying division problems involving fractions, whole numbers, and mixed numbers should be to **change all whole and mixed numbers to improper fractions**.

SAMPLE PROBLEM 7 (NEXT PAGE) DEMONSTRATES DIVISION INVOLVING FRACTIONS, WHOLE NUMBERS, AND MIXED NUMBERS.

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SAMPLE PROBLEM 6 WITH SOLUTION

The Problem:

Divide & Reduce: $^{5}/_{6} \div {}^{3}/_{4}$

The Solution:

Remember: Division is defined to be multiplying by the reciprocal. (Reciprocals are numbers that when multiplied together equal 1.) **Thus, to multiply fractions we invert** (flip) **the number we are dividing by, then multiply**.

1. Invert (flip) the number you are dividing by

$$^{5}/_{6} * {}^{4}/_{3} =$$

2. Cancel any factors in the numerators that also appear in the denominators.

$$^{5}/_{\cancel{6}_{3}} * {}^{\cancel{4}^{2}}/_{3} =$$

3. Multiply the resulting numerators together (5 * 2 = 10).
4. Multiply the resulting denominators together (3 * 3 = 9).
5. Write the resulting fraction or mixed number: $^{10}/_{9} = 1\,{}^{1}/_{9}$.

A. Cancel where possible, then divide and reduce. Refer to Sample Problem 6 as needed. For more practice, see pages 47 and 48 of Contemporary's Number Power 2 work-text.

1. $^{9}/_{10} \div {}^{7}/_{15} =$
2. $^{5}/_{8} \div {}^{3}/_{2} =$
3. $^{15}/_{4} \div {}^{3}/_{8} =$

SAMPLE PROBLEM 7 WITH SOLUTION

The Problem:

Divide & Reduce, canceling when possible: $10 \div 2\,{}^{1}/_{7}$

The Solution:

1. Convert all whole numbers and mixed numbers to improper fractions.

$$10 \div 2\,{}^{1}/_{7} =$$
$$^{10}/_{1} \div {}^{15}/_{7} =$$

2. Invert (flip) the number you are dividing by

$$^{10}/_{1} * {}^{7}/_{15} =$$

3. Cancel any factors in the numerators that also appear in the denominators.

$$^{\cancel{10}^{2}}/_{1} * {}^{7}/_{\cancel{15}_{3}} =$$

4. Multiply the resulting numerators together (2 * 7 = 14).
5. Multiply the resulting denominators together (1 * 3 = 3).
6. Write the resulting fraction or mixed number: $^{14}/_{3} = 4\,{}^{2}/_{3}$.

B. Convert whole numbers and mixed numbers to improper fractions, cancel where possible, then divide and reduce. Refer to Sample Problem 7 as needed. For more practice, see pages 49 – 54 of Contemporary's Number Power 2 work-text.

1. $15 \div {}^{5}/_{6} =$
2. $^{3}/_{8} \div 3 =$
3. $8\,{}^{1}/_{3} \div 1\,{}^{1}/_{4} =$

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(Multiplying and Dividing Fractions)

IV. Solving Multiplication & Division problems involving fractions, whole numbers, and mixed nummbers

Sample Problem 8 demonstrates how to solve a multiplication problem involving fractions.

Sample Problem 8 with Solution

The Problem:

Joan wants to know how many canning jars she will need to store the 12 pounds of peppers she has prepared. She estimates that each jar will hold $^{2}/_{3}$ of a pound. How many jars does she need?

The Solution:

1. We are to find how many $^{2}/_{3}$-pound jars Joan needs to can 12 pounds of peppers.

2. A good way to decide what operation(s) to use in solving a problem is to simplify the problem by rounding all fractions and mixed numbers to whole numbers and deciding how you would solve the simpler problem.

   If we round $^{2}/_{3}$ to 1, our problem would simplify to: "How many 1-pound jars are needed to store 12 pounds of peppers." Now, it is easy to see that we must divide 12 by 1 to solve our simplified problem. Thus, to solve the original problem we must divide 12 by $^{2}/_{3}$. (We estimate that we will need somewhat more than 12 jars.)

   The calculation below gives the exact number of jars needed:

$$12 \div {}^{2}/_{3} = {}^{12}/_{1} \div {}^{2}/_{3} = {}^{\overset{6}{\cancel{12}}}/_{1} * {}^{3}/_{\underset{1}{\cancel{2}}} = 18$$

3. Joan needs 18 jars to can the 12 pounds of peppers.

A. Completely solve the problems below using steps (*i*), (*ii*), and (*iii*). Sample Problem 8 is provided as a model. Additional practice problems can be found on pages 44 – 46 and 55 – 57 of Contemporary's Number Power 2 work-text. You may also want to review the Introduction to Problem Solving handout.

i. State what it is you are to find. Give your answer as a complete sentence.
ii. Solve the problem, showing your work.
iii. State the answer in a complete sentence.

1. A recipe that serves eight calls for $2^{1}/_{2}$ cups of milk. How much milk should you use if you need to adjust the recipe to serve ten?

2. How many pieces of shelving $1^{1}/_{3}$ feet long can be cut from an 8-foot board?

3. If it takes $1^{1}/_{3}$ yards of material to make one skirt, how many skirts can be made from 12 yards of material?

4. Harvey & Louise are planning a trip to her parents for Christmas. They estimate the total trip mileage at 840 miles and plan to make overnight stops at the one-third and two-thirds points of the trip. How far will they have traveled when they make their second stop?

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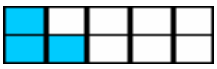
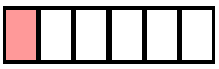
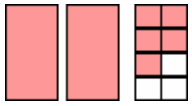
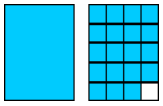
ANSWER KEY

SECTION I: REVIEW PROBLEMS

A1:  A2:  A3: 30 A4: $^{7}/_{15}$ A5: $^{2}/_{15}$

B1:  B2:  B3: 40 B4: $4^{23}/_{40}$ B5: $^{27}/_{40}$

C1: $2^{28}/_{45}$ C2: $4^{19}/_{55}$ C3: $8^{11}/_{20}$ C4: $6^{11}/_{16}$ C5: $16^{2}/_{15}$ C6: $3^{5}/_{8}$

D1: $1^{1}/_{4}$ in. D2: $4^{5}/_{16}$ in.

E1: $34^{5}/_{8}$ points E2: $10^{17}/_{20}$ miles E3: $^{61}/_{80}$ of income E4: $^{19}/_{80}$ of income

SECTION II: MULTIPLYING FRACTIONS & MIXED NUMBERS

A1: $^{63}/_{130}$ A2: $^{1}/_{8}$ A3: $^{15}/_{16}$ B1: $^{1}/_{8}$ B2: $^{1}/_{2}$ B3: $2^{1}/_{2}$

C1: $2^{2}/_{3}$ C2: $3^{3}/_{16}$ C3: $46^{9}/_{80}$

SECTION III: DIVIDING FRACTIONS & MIXED NUMBERS

A1: $1^{13}/_{14}$ A2: $^{5}/_{12}$ A3: 10 B1: 18 B2: $^{1}/_{8}$ B3: $6^{2}/_{3}$

SECTION IV: SOLVING MULTIPLICATION & DIVISION PROBLEMS INVOLVING FRACTIONS, WHOLE NUMBERS, & MIXED NUMBERS

A1: $3^{1}/_{8}$ cups A2: 6 pieces A3: 9 skirts A4: 560 miles